Differential Geometry I Fall 2013 Eth Zurich

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Differential Geometry American Mathematical Soc.
Differential Geometry is a wide field. We have chosen to concentrate upon certain aspects that are appropriate for an introduction to the subject; we have not attempted an encyclopedic treatment. Book II deals with more advanced material than Book I and is aimed at the graduate level. Chapter 4 deals with additional topics in Riemannian geometry. Properties of real analytic curves given by a single ODE and of surfaces given by a pair of ODEs are studied, and the volume of geodesic balls is treated. An introduction to both holomorphic and Kähler geometry is given. In Chapter 5, the basic properties of de Rham cohomology are discussed, the Hodge Decomposition Theorem, Poincaré duality, and the Künneth formula are proved, and a brief introduction to the theory of characteristic classes is given. In Chapter 6, Lie groups and Lie algebras are dealt with. The exponential map, the classical groups, and geodesics in the context of a bi-invariant metric are discussed. The de Rham cohomology of compact Lie groups and the Peter--Weyl Theorem are treated. In Chapter 7, material concerning homogeneous spaces and symmetric spaces is presented. Book II concludes in Chapter 8 where the relationship between simplicial cohomology, singular cohomology, sheaf cohomology, and de Rham cohomology is established. We have given some different proofs than those that are classically given and there is some new material in these volumes. For example, the treatment of the total curvature and length of curves given by a single ODE is new as is the discussion of the total Gaussian curvature of a surface defined by a pair of ODEs.
Recent Advances in the Geometry of Submanifolds MIT Press
This text presents a graduate-level introduction to differential geometry for mathematics and physics students. The exposition follows the historical development of the concepts of connection and curvature with the goal of explaining the Chern–Weil theory of characteristic classes on a principal bundle. Along the way we encounter some of the high points in the history of differential geometry, for example, Gauss' Theorema Egregium and the Gauss–Bonnet theorem. Exercises throughout the book test the reader's understanding of the material and sometimes illustrate extensions of the theory. Initially, the prerequisites for the reader include a passing familiarity with manifolds. After the first chapter, it becomes necessary to understand and manipulate differential forms. A knowledge of de Rham cohomology is required for the last third of the text. Prerequisite material is contained in author's text An Introduction to Manifolds, and can be learned in one semester. For the benefit of the reader and to establish common notations, Appendix A recalls the basics of manifold theory. Additionally, in an attempt to make the exposition more self-contained, sections on algebraic constructions such as the tensor product and the exterior power are included. Differential geometry, as its name implies, is the study of geometry using differential calculus. It dates back to Newton and Leibniz in the seventeenth century, but it was not until the nineteenth century, with the work of Gauss on surfaces and Riemann on the curvature tensor, that differential geometry flourished and its modern foundation was laid. Over the past one hundred years, differential geometry has proven indispensable to an understanding of the physical world, in Einstein's general theory of relativity, in the theory of gravitation, in gauge theory, and now in

string theory. Differential geometry is also useful in topology, several complex variables, algebraic geometry, complex manifolds, and dynamical systems, among other fields. The field has even found applications to group theory as in Gromov's work and to probability theory as in Diaconis's work. It is not too far-fetched to argue that differential geometry should be in every mathematician's arsenal.

Aspects of Differential Geometry IV Courier Corporation

Starting from an undergraduate level, this book systematically develops the basics of • Calculus on manifolds, vector bundles, vector fields and differential forms, • Lie groups and Lie group actions, • Linear symplectic algebra and symplectic geometry, • Hamiltonian systems, symmetries and reduction, integrable systems and Hamilton-Jacobi theory. The topics listed under the first item are relevant for virtually all areas of mathematical physics. The second and third items constitute the link between abstract calculus and the theory of Hamiltonian systems. The last item provides an introduction to various aspects of this theory, including Morse families, the Maslov class and caustics. The book guides the reader from elementary differential geometry to advanced topics in the theory of Hamiltonian systems with the aim of making current research literature accessible. The style is that of a mathematical textbook,with full proofs given in the text or as exercises. The material is illustrated by numerous detailed examples, some of which are taken up several times for demonstrating how the methods evolve and interact.

Aspects of Differential Geometry V American Mathematical Soc.

Differential Manifolds is a modern graduate-level introduction to the important field of differential topology. The concepts of differential topology lie at the heart of many mathematical disciplines such as differential geometry and the theory of lie groups. The book introduces both the h-cobordism theorem and the classification of differential structures on spheres. The presentation of a number of topics in a clear and simple fashion make this book an outstanding choice for a graduate course in differential topology as well as for individual study. Presents the study and classification of smooth structures on manifolds It begins with the elements of theory and concludes with an introduction to the method of surgery Chapters 1-5 contain a detailed presentation of the foundations of differential topology--no knowledge of algebraic topology is required for this self-contained section Chapters 6-8 begin by explaining the joining of manifolds along submanifolds, and ends with the proof of the h-cobordism theory Chapter 9 presents the Pontriagrin construction, the principle link between differential topology and homotopy theory; The final chapter introduces the method of surgery and applies it to the classification of smooth structures on spheres

Differential Geometry Academic Press

This outstanding guide supplies important mathematical tools for diverse engineering applications, offering engineers the basic concepts and terminology of modern global differential geometry. Suitable for independent study as well as a supplementary text for advanced undergraduate and graduate courses, this volume also constitutes a valuable reference for control, systems, aeronautical, electrical, and mechanical engineers. The treatment's ideas are applied mainly as an introduction to the Lie theory of differential equations and to examine the role of Grassmannians in control systems analysis. Additional topics include the fundamental notions of manifolds, tangent spaces, vector fields, exterior algebra, and Lie algebras. An appendix reviews concepts related to vector calculus, including open and closed sets, compactness, continuity, and derivative.

Aspects of Differential Geometry III Springer Science & Business Media

An introductory textbook on the differential geometry of curves and surfaces in 3-dimensional Euclidean space, presented in its simplest, most essential form. With problems and solutions. Includes 99 illustrations.

Topics in Differential Geometry Cambridge University Press

This book surveys the differential geometry of varieties with degenerate Gauss maps, using moving frames and exterior differential forms as well as tensor methods. The authors illustrate the structure of varieties with degenerate Gauss maps, determine the singular points and singular varieties, find focal images and construct a classification of the varieties with degenerate Gauss maps.

Differential Forms and Connections American Mathematical Soc.

This book contains a series of papers on some of the longstanding research problems of geometry, calculus of variations, and their applications. It is suitable for advanced graduate students, teachers, research mathematicians, and other professionals in mathematics.

A First Course in Differential Geometry American Mathematical Soc.

An explanation of the mathematics needed as a foundation for a deep understanding of general relativity or quantum field theory. Physics is naturally expressed in mathematical language. Students new to the subject must simultaneously learn an idiomatic mathematical language and the content that is expressed in that language. It is as if they were asked to read Les Misérables while struggling with French grammar. This book offers an innovative way to learn the differential geometry needed as a foundation for a deep understanding of general relativity or quantum field theory as taught at the college level. The approach taken by the authors (and used in their classes at MIT for many years) differs from the conventional one in several ways, including an emphasis on the development of the covariant derivative and an avoidance of the use of traditional index notation for tensors in favor of a semantically richer language of vector fields and differential forms. But the biggest single difference is the authors' integration of computer programming into their explanations. By programming a computer to interpret a formula, the student soon learns whether or not a formula is correct. Students are led to improve their program, and as a result improve their understanding.

Differential Geometry of Curves and Surfaces CRC Press

Differential Geometry of Curves and Surfaces, Second Edition takes both an analytical/theoretical approach and a visual/intuitive approach to the local and global properties of curves and surfaces. Requiring only multivariable calculus and linear algebra, it develops students' geometric intuition through interactive computer graphics applets suppor

Metric Structures in Differential Geometry University of Toronto Press

This volume contains the proceedings of the AMS Special Session on Geometry of Submanifolds, held from October 25–26, 2014, at San Francisco State University, San Francisco, CA, and the AMS Special Session on Recent Advances in the Geometry of Submanifolds: Dedicated to the Memory of Franki Dillen (1963–2013), held from March 14–15, 2015, at Michigan State University, East Lansing, MI. The focus of the volume is on recent studies of submanifolds of Riemannian, semi-Riemannian, Kaehlerian and contact manifolds. Some of these use techniques in classical differential geometry, while others use methods from ordinary differential equations, geometric analysis, or geometric PDEs. By brainstorming on the fundamental problems and exploring a large variety of questions studied in submanifold geometry, the editors hope to provide mathematicians with a working tool, not just a collection of individual contributions. This volume is dedicated to the memory of Franki Dillen, whose work in submanifold theory attracted the attention of and inspired many geometers.

Ergodic Theory and Negative Curvature American Mathematical Soc.

This text contains an elementary introduction to continuous groups and differential invariants; an extensive treatment of groups of motions in euclidean, affine, and riemannian geometry; more. Includes exercises and 62 figures.

Aspects of Differential Geometry I Springer Science & Business Media

Book V completes the discussion of the first four books by treating in some detail the analytic results in elliptic operator theory used previously. Chapters 16 and 17 provide a treatment of the techniques in Hilbert space, the Fourier transform, and elliptic operator theory necessary to establish the spectral decomposition theorem of a self-adjoint operator of Laplace type and to prove the Hodge Decomposition Theorem that was stated without proof in Book II. In Chapter 18, we treat the de Rham complex and the Dolbeault complex, and discuss spinors. In Chapter 19, we discuss complex geometry and establish the Kodaira Embedding Theorem.

Nonlinear Partial Differential Equations in Differential Geometry CRC Press

Book IV continues the discussion begun in the first three volumes. Although it is aimed at first-year graduate students, it is also intended to serve as a basic reference for people working in affine differential geometry. It also should be accessible to undergraduates interested in affine differential geometry. We are primarily concerned with the study of affine surfaces which are locally homogeneous. We discuss affine gradient Ricci solitons, affine Killing vector fields, and geodesic completeness. Opozda has classified the affine surface geometries which are locally homogeneous; we follow her classification. Up to isomorphism, there are two simply connected Lie groups of dimension 2. The translation group R2 is Abelian and the ???? + ?? group is non-Abelian. The first chapter presents foundational material. The second chapter deals with Type ?? surfaces. These are the left-invariant affine geometries on R2. Associating to each Type ?? surface the space of solutions to the quasi-Einstein equation corresponding to the eigenvalue ?? = -1 turns out to be a very powerful technique and plays a central role in our study as it links an analytic invariant with the underlying geometry of the surface. The third chapter deals with Type ?? surfaces; these are the left-invariant affine geometries on the ???? + ?? group. These geometries form a very rich family which is only partially understood. The only remaining homogeneous geometry is that of the sphere ??2. The fourth chapter presents relations between the geometry of an affine surface and the geometry of the cotangent bundle equipped with the neutral signature metric of the modified Riemannian extension.

Differential Geometry of Varieties with Degenerate Gauss Maps International Press of Boston

This book examines the exciting interface between differential geometry and continuum mechanics, now recognised as being of increasing technological significance. Topics discussed include isometric embeddings in differential geometry and the relation with microstructure in nonlinear elasticity, the use of manifolds in the description of microstructure in continuum mechanics, experimental measurement of microstructure, defects, dislocations, surface energies, and nematic liquid crystals. Compensated compactness in partial differential equations is also treated. The volume is intended for specialists and non-specialists in pure and applied geometry, continuum mechanics, theoretical physics, materials and engineering sciences, and partial differential equations. It will also be of interest to postdoctoral scientists and advanced postgraduate research students. These proceedings include revised written versions of the majority of papers presented by leading experts at the ICMS Edinburgh Workshop on Differential Geometry and Continuum Mechanics held in June 2013. All papers have been peer reviewed.

Recent Advances in the Geometry of Submanifolds Springer

The uniqueness of this text in combining geometric topology and differential geometry lies in its unifying thread: the notion of a surface. With numerous illustrations, exercises and examples, the student comes to understand the relationship of the modern abstract approach to geometric intuition. The text is kept at a concrete level, avoiding unnecessary abstractions, yet never sacrificing mathematical rigor. The book includes topics not usually found in a single book at this level.

Differential Geometry, Calculus of Variations, and Their Applications Cambridge University Press

Curves and surfaces are objects that everyone can see, and many of the questions that can be asked about them are natural and easily understood. Differential geometry is concerned with the precise mathematical formulation of some of these questions, while trying to answer them using calculus techniques. The geometry of differentiable manifolds with structures is one of the most important branches of modern differential geometry. This well-written book discusses the theory of differential and Riemannian manifolds to help students understand the basic structures and consequent developments. While introducing concepts such as bundles, exterior algebra and calculus, Lie group and its algebra and calculus, Riemannian geometry, submanifolds and hypersurfaces, almost complex manifolds, etc., enough care has been taken to provide necessary details which enable the reader to grasp them easily. The material of this book has been successfully tried in classroom teaching. The book is designed for the postgraduate students of Mathematics. It will also be useful to the researchers working in the field of differential geometry and its applications to general theory of relativity and cosmology, and other applied areas. KEY FEATURES Provides basic concepts in an easy-to-understand style. Presents the subject in a natural way. Follows a coordinate-free approach. Includes a large number of solved examples and illuminating illustrations. Gives notes and remarks at appropriate places.

Introduction to Differential Geometry with Applications to Navier-Stokes Dynamics
Courier Corporation

Focussing on the mathematics related to the recent proof of ergodicity of the (Weil–Petersson) geodesic flow on a nonpositively curved space whose points are negatively curved metrics on surfaces, this book provides a broad introduction to an important current area of research. It offers original textbook-level material suitable for introductory or advanced courses as well as deep insights into the state of the art of the field, making it useful as a reference and for self-study. The first chapters introduce hyperbolic dynamics, ergodic theory and geodesic and horocycle flows, and include an English translation of Hadamard's original proof of the Stable-Manifold Theorem. An outline of the strategy, motivation and context behind the ergodicity proof is followed by a careful exposition of it (using the Hopf argument) and of the pertinent context of Teichmüller theory. Finally, some complementary lectures describe the deep connections between geodesic flows in negative curvature and Diophantine approximation.

Differential Geometry Morgan & Claypool Publishers

Accessible, concise, and self-contained, this book offers an outstanding introduction to three related subjects: differential geometry, differential topology, and dynamical systems. Topics of special interest addressed in the book include Brouwer's fixed point theorem, Morse Theory, and the geodesic flow. Smooth manifolds, Riemannian metrics, affine connections, the curvature tensor, differential forms, and integration on manifolds provide the foundation for many applications in dynamical systems and mechanics. The authors also discuss the Gauss-Bonnet theorem and its implications in non-Euclidean geometry models. The differential topology aspect of the book centers on classical, transversality theory, Sard's theorem, intersection theory, and fixed-point theorems. The construction of the de Rham cohomology builds further arguments for the strong connection between the differential structure and the topological structure. It also furnishes some of the tools necessary for a complete understanding of the Morse theory. These discussions are followed by an introduction to the theory of hyperbolic systems, with emphasis on the quintessential role of the geodesic flow. The integration of geometric theory, topological theory, and concrete applications to dynamical systems set this book apart. With clean, clear prose and effective examples, the authors' intuitive approach creates a treatment that is comprehensible to relative beginners, yet rigorous enough for those with more background and experience in the field.

Aspects of Differential Geometry II Allied Publishers

Differential Geometry is a wide field. We have chosen to concentrate upon certain aspects that are appropriate for an introduction to the subject; we have not attempted an encyclopedic treatment. In Book I, we focus on preliminaries. Chapter 1 provides an introduction to multivariable calculus and treats the Inverse Function Theorem, Implicit Function Theorem, the theory of the Riemann Integral, and the Change of Variable Theorem. Chapter 2 treats smooth manifolds, the tangent and cotangent bundles, and Stokes' Theorem. Chapter 3 is an introduction to Riemannian geometry. The Levi-Civita connection is presented, geodesics introduced, the Jacobi operator is discussed, and the Gauss-Bonnet Theorem is proved. The material is appropriate for an undergraduate course in the subject. We have given some different proofs than those that are classically given and there is some new material in these volumes. For example, the treatment of the Chern-Gauss-Bonnet Theorem for pseudo-Riemannian manifolds with boundary is new.